SYMBOLIC COMPUTATION TECHNIQUES USING MAPLE

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Abstract

This paper deals with Maple as a symbolic computation techniques. After introducing some capabilities of Maple, some examples have been discussed to illustrate the Maple. A teaching model at a University is given within the conclusions.

Introduction

Maple is a symbolic manipulation package with indefinite accuracy arithmetic to which a graphics capability has been added on. The Maple is a very wide ranging tool. The capabilities range from basic algebra and calculus to projective geometry, group theory and number theory and more. There is also a programming language that allows all the expected things such as branching, looping and procedures. The Maple is used both in University Education as well as in Industry. It also is very useful in research and development projects.

For practice, a worksheet will be used which has an easily read output and is easy to edit. Expressions may be modified and results recalculated with a few steps so that exploration and experiment are simple with the ability to change commands. It is a simple matter to put explanatory text between the commands and whole may be saved. A help system has been integrated with a click-and-see browser which moves down the branches of a tree. At each level a list of nodes is displayed. Clicking on one of them will display the corresponding list of subnodes. The result of the commands is a window containing the verbatim extract from the Library Reference Manual.

The Maple package includes:

- Basic operations of algebra
- Fourier and Laplace transforms
- Differential equations (symbolic and numeric)
- Linear algebra functions
- Complex functions
- Plots (function, contour plots, vector field plot, animated plots …)
- Special packages (padic, numerical approximation, network, …)

In addition to the existing relevant courses in mathematics and in computer science, the following new courses could be prepared:

Undergraduate course:
Contents: Generalities of programs for Symbolic Computations; Mathematical manipulations (Computer Algebra); Differentiations; Integrations; Differential Equations; Polynomials; Solving systems.

Graduate course:
Contents: Automatic differentiation; Automatic integration; Partial differential equations; Complex analysis

Research Projects and Seminars: Special research domain such as computational conformal geometry, computer graphics, Computer Visualisation

Applications

Example : Computer Algebra

```
expand(y*(x-a)*(x-b)*(x-c));
```

$$y x^3 - y x^2 c - y x^2 b + y x b c - y a x^2 + y a x c + y a b x - y a b c$$

```
divide(-x^5+9*x^4-18*x^3+x^2+19*x-10,x-2,'q'); #Polynomial division, 'q' stores the quotient
```

$$true$$

```
q; #To show the result stored in 'q'
```

$$-x^4 + 7x^3 - 4x^2 - 7x + 5$$

```
pol1:=-x^5+9*x^4-18*x^3-x^2+19*x-10;
```

$$pol1 := -x^5 + 9x^4 - 18x^3 - x^2 + 19x - 10$$

```
pol2:=x-2;
```

$$pol2 := x - 2$$

```
divide(pol1,pol2,'q');
```

$$false$$

```
q;
```

$$-x^4 + 7x^3 - 4x^2 - 7x + 5$$

```
solve(a*x^2-b*x+c,x);
```

$$\frac{b + \sqrt{b^2 - 4ac}}{2a}, \frac{b - \sqrt{b^2 - 4ac}}{2a}$$

```
solve(a*x^2-b*x+c,{x});
```

$$\{x = \frac{b + \sqrt{b^2 - 4ac}}{2a}\}, \{x = \frac{b - \sqrt{b^2 - 4ac}}{2a}\}$$

Differentiation

```
restart;
D(x^2);
```

$$2\,\mathrm{D}(x)\,x$$

```
diff(x^2,x);
```

$$2x$$

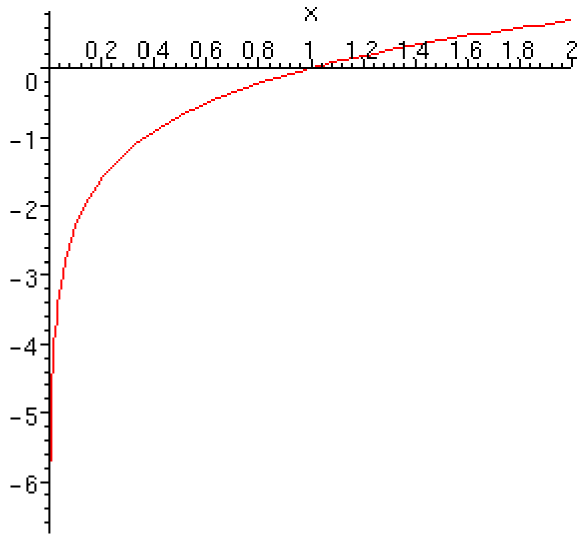
```
f:=x -> ln(x);
```

$$f := \ln$$

```
fp:=x -> diff(f(x),x);
```

$$fp := x \rightarrow \mathrm{diff}(\mathrm{f}(x), x)$$

```
plot(f(x),x=0..2);
```



Differential Equations

The most commonly used command for investigating the behaviour of ODEs in MAPLE is dsolve.

This is used both closed form and numerical solutions.

dsolve(eq, var)

```
eq := diff(v(t), t) + a*t =0;
```

$$eq := \left(\frac{d}{dt}\mathrm{v}(t)\right) + a\,t = 0$$

initial values

```
ini := v(1) = b;
```

$$ini := \mathrm{v}(1) = b$$

use dsolve(..)

```
dsolve({eq, ini}, {v(t)});
```

$$\mathrm{v}(t) = -\frac{1}{2} a t^2 + \frac{1}{2} a + b$$

Definition of the 1.ODE

ode1 :=diff(y(t), t$2) + sin(t)^2*diff(y(t),t) + y(t) = cos(t)^2;

$$odel := \left(\frac{d^2}{dt^2}\mathrm{y}(t)\right) + \sin(t)^2\left(\frac{d}{dt}\mathrm{y}(t)\right) + \mathrm{y}(t) = \cos(t)^2$$

with(DEtools):

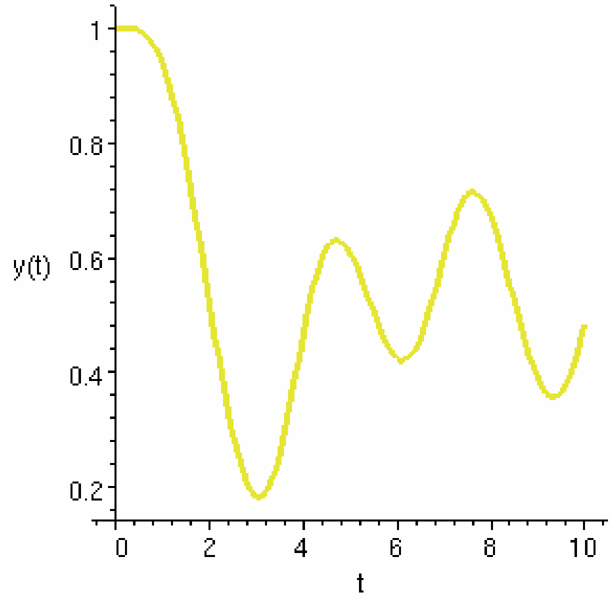
ic1 := y(0) = 1, D(y)(0) = 0;

$$icl := \mathrm{y}(0) = 1, \mathrm{D}(y)(0) = 0$$

dsolve({ode1, ic1}, {y(t)});

The Plot of ODE: *DEplot(ode, dep-var, range, [ini-conds])*

DEplot(ode1, y(t), 0..10, [[ic1]], stepsize=0.1);



eq1 := diff(y(t),t) + y(t) + x(t) = 0;

$$eql := \left(\frac{d}{dt}\mathrm{y}(t)\right) + \mathrm{y}(t) + \mathrm{x}(t) = 0$$

eq2 := y(t) = diff(x(t), t);

$$eq2 := \mathrm{y}(t) = \frac{d}{dt}\mathrm{x}(t)$$

ini1 := x(0) = 0, y(0)=2;

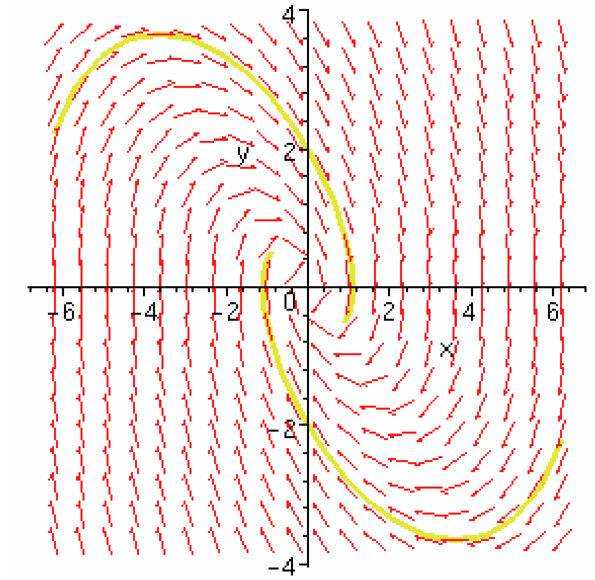
$$inil := \mathrm{x}(0) = 0, \mathrm{y}(0) = 2$$

ini2 := x(0)=0, y(0)=-2;

$$ini2 := \mathrm{x}(0) = 0, \mathrm{y}(0) = -2$$

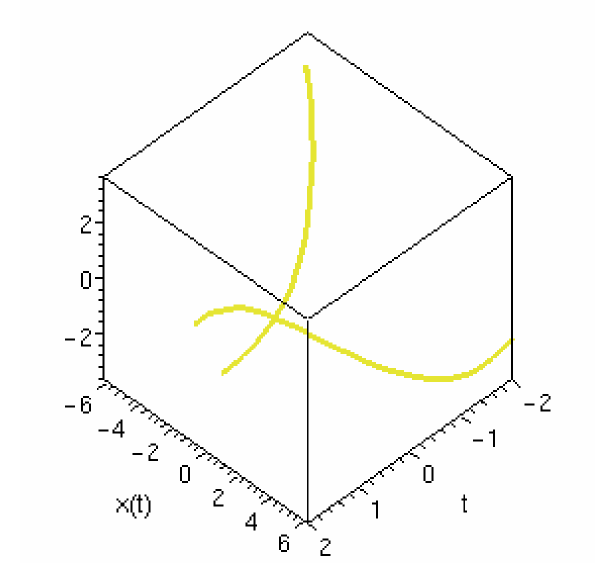
System of equations

DEplot({eq1, eq2}, [x(t), y(t)], -2..2,[[ini1], [ini2]]);



3_D plot

DEplot3d({eq1, eq2}, [x(t), y(t)], -2..2, [[ini1], [ini2]]);



Partial Differential Equations : wave equation 1-D

pdsolve(pde, var)

wave := diff(u(x,t),t,t) - c^2*diff(u(x,t), x,x);

$$wave := \left(\frac{\partial^2}{\partial t^2}\mathrm{u}(x, t)\right) - c^2\left(\frac{\partial^2}{\partial x^2}\mathrm{u}(x, t)\right)$$

load the PDEtools

```
with(PDEtools):
sol := pdsolve( wave, u(x,t));
```

$$sol := u(x,t) = \_F1(c\,t + x) + \_F2(c\,t - x)$$

```
f1 := xi -> exp(-xi^2);
```

$$f1 := \xi \rightarrow e^{(-\xi^2)}$$

```
f2 := xi -> piecewise(-1/2<xi and xi<1/2, 1, 0);
```

$$f2 := \xi \rightarrow \mathrm{piecewise}\left(\frac{-1}{2} < \xi \text{ and } \xi < \frac{1}{2}, 1, 0\right)$$

substitute into the solution

```
eval(sol, {_F1=f1, _F2=f2, c=1});
```

$$u(x,t) = e^{(-(t+x)^2)} + \left(\begin{cases} 1 & -t + x < \frac{1}{2} \text{ and } t - x < \frac{1}{2} \\ 0 & otherwise \end{cases}\right)$$

select the solution

```
rhs(%);
```

$$e^{(-(t+x)^2)} + \left(\begin{cases} 1 & -t + x < \frac{1}{2} \text{ and } t - x < \frac{1}{2} \\ 0 & otherwise \end{cases}\right)$$

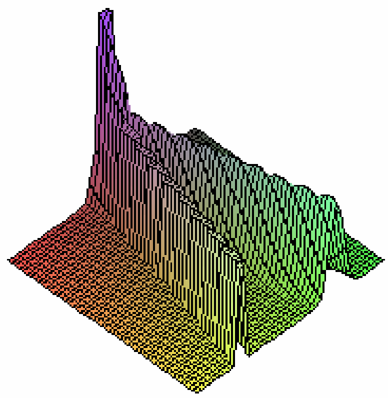
convert the expression to a function

```
f := unapply(%, x, t);
```

$$f := (x,t) \rightarrow e^{(-(t+x)^2)} + \mathrm{piecewise}\left(-t + x < \frac{1}{2} \text{ and } t - x < \frac{1}{2}, 1, 0\right)$$

3-D plot

```
plot3d(f, -10..10, 0..5, grid=[40,50]);
```



Conclusions

Symbolic computation techniques could be implemented, in addition to the existing relevant courses in mathematics and in computer science, using the possibilities of Maple. In an **Undergraduate course could be handled:** Generalities of programs for Symbolic Computations; Mathematical manipulations (Computer Algebra); Differentiations; Integrations; Differential Equations; Polynomials; Solving systems.

In a Graduate course can be discussed: Automatic differentiation; Automatic integration; Partial differential equations; Complex analysis.

Special research domain such as computational conformal geometry, computer graphics, Computer Visualisation can use Maple in **Research Projects and Seminars**.

References

[1] A. Heck Introduction to Maple, Springer, 1996

[2] B. Buchberger, et all. Computer Algebra – Symbolic and Algebraic Computation, Springer, 1983

[3] Maple 9, Learning Guide, Maplesoft, 2003